OPTIMIZATION OF BODY SHAPE AT SMALL REYNOLDS NUMBERS

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The problem of the optimization of the shape of a body in a stream of viscous liquid or gas was treated in [1-5]. The necessary conditions for a body to offer minimum resistance to the flow of a viscous gas past it were derived in [1]. The necessary optimality conditions when the motion of the fluid is described by the approximate Stokes equations were derived in [2]. The shape of a body of minimum resistance was found numerically in [3] in the Stokes approximation. The optimality conditions when the motion of the fluid is described by the Navier–Stokes equations were derived in [4, 5], and in [4] these conditions were extended to the case of a fluid whose motion is described in the boundary-layer approximation. The necessary optimality conditions when the motion of the fluid is described by the approximate Oseen equations were derived in [5] and an asymptotic analysis of the behavior of the optimum shape near the critical points was performed for arbitrary Reynolds numbers.

§1. The boundary-value problem for determining the shape of a body of minimum resistance among bodies of a given volume formulated in [4, 5] can be reduced to the form

$$\begin{gathered}\Delta \mathbf{v} - \nabla p = \mathrm{Re}(\mathbf{v}\nabla)\mathbf{v},\ \nabla\mathbf{v} = 0,\ (\mathbf{v})_S = 0,\ (\mathbf{v})_\Sigma = \mathbf{v}_\Sigma,\\ \Delta\mathbf{u} - \nabla q = \mathrm{Re}[\mathbf{v}\nabla\mathbf{v} - (\mathbf{v}\nabla)\mathbf{u}],\ \nabla\mathbf{u} = 0,\\ (\mathbf{u})_S = 0,\ (\mathbf{u})_\Sigma = \mathbf{v}_\Sigma,\ (\boldsymbol{\Omega}\boldsymbol{\Omega}^*)_S = \mathrm{const},\end{gathered} \quad (1.1)$$

where $\mathbf{v}$ and p are, respectively, the velocity and pressure fields in the stream of fluid; u and q are certain auxiliary vector and scalar functions, S is the surface of the optimum body; Σ is the outer boundary of the volume of fluid considered on which the velocity distribution $\mathbf{v}_\Sigma$ is specified; $\Omega = \mathrm{rot}\ \mathbf{v}$, $\Omega^* = \mathrm{rot}\ \mathbf{u}$. Suppose the surface S is described by the parametric equations $x_i = x_i(r, t)$. Since the optimization problem is solved for the isoperimetric condition of constant volume, the functions $x_i(r, t)$ must satisfy the equation

$$\int_S n_i x_i(r, t)\, dS = 1,$$

where the n_i are the components of the outward normal to surface S.

The boundary-value problem (1.1) depends on the Reynolds number Re and, consequently, the shape of the optimum body also depends on the Reynolds number. Suppose the surface S_0 of the body which is optimum in the Stokes approximation (Re = 0) is described by the equations $x_i = x_{0i}(r, t)$. We assume that the equation of the surface of the body S_{Re} which is optimum for a nonzero Reynolds number can be written in the form

$$x_i = x_i(r, t, \mathrm{Re}) = x_{0i}(r, t) + n_i[\mathrm{Re} f_1(r, t) + \mathrm{Re}^2 f_2(r, t) + \ldots]. \quad (1.2)$$

The expansion (1.2) is possible when the surface S_0 is smooth. If there are critical points (branch points of the streamlines) on the surface S_0, the surface S_0 in the neighborhood of these points has the shape of a cone with a vertex angle of 120° [5]. If the surface determined by Eqs. (1.2) is a cone with a vertex angle of 120° it is shown in [5] that the equations, boundary conditions, and optimality conditions in the neighborhood of a critical point will be satisfied to an accuracy of $O(\mathrm{Re}^4 f_1^3(r_0, t_0))$, where r_0 and t_0 are the values of the parameters r and t corresponding to the critical point.

Suppose the functions $\mathbf{v}$, p, u, and q satisfy the boundary-value problem (1.1) with the boundary conditions specified on the surface S_{Re}. We expand these functions in powers of Re

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$$\mathbf{v} = \mathbf{v}_0 + \mathrm{Re}\mathbf{v}_1 + \mathrm{Re}^2\mathbf{v}_2 + \ldots,\ p = p_0 + \mathrm{Re}p_1 + \mathrm{Re}^2p_2 + \ldots,$$
$$\mathbf{u} = \mathbf{u}_0 + \mathrm{Re}\mathbf{u}_1 + \mathrm{Re}^2\mathbf{u}_2 + \ldots,\ q = q_0 + \mathrm{Re}q_1 + \mathrm{Re}^2q_2 + \ldots \tag{1.3}$$

Substituting expansions (1.3) into the boundary-value problem (1.1), moving the boundary conditions from surface S_{Re} onto surface S_0, taking account of (1.2), and expanding the isoperimetric condition in powers of Re, we obtain a sequence of boundary-value problems for determining the functions f_i, $\mathbf{v}_i$, p_i, $\mathbf{u}_i$, and q_i. In the zero approximation we have

$$\begin{gathered}\Delta\mathbf{v}_0 - \nabla p_0 = 0,\ \nabla\mathbf{v}_0 = 0,\ (\mathbf{v}_0)_{S_0} = 0,\ (\mathbf{v}_0)_\Sigma = \mathbf{v}_\Sigma,\\ \Delta\mathbf{u}_0 - \nabla q_0 = 0,\ \nabla\mathbf{u}_0 = 0,\ (\mathbf{u}_0)_{S_0} = 0,\ (\mathbf{u}_0)_\Sigma = \mathbf{v}_\Sigma,\\ (\mathbf{\Omega}_0\mathbf{\Omega}_0^*)_{S_0} = C_0,\ \int\limits_{S_0} x_{0i}n_i dS = 1\end{gathered} \tag{1.4}$$

where the constant C_0 is determined from the isoperimetric condition. The boundary-value problems for the functions $\mathbf{v}_0$, p_0 and $\mathbf{u}_0$, q_0 are the same and therefore $\mathbf{u}_0 = \mathbf{v}_0$, $q_0 = p_0 +$ const, and $\mathbf{\Omega}_0 = \mathbf{\Omega}_0^*$. In this case the boundary-value problem is equivalent to the problem formulated in [2] for the Stokes approximation.

For the first approximation in Re problem (1.1) is reduced to the form

$$\begin{gathered}\Delta\mathbf{v}_1 - \nabla p_1 = (\mathbf{v}_0\nabla)\mathbf{v}_0,\ \nabla\mathbf{v}_1 = 0,\\ \Delta\mathbf{u}_1 - \nabla q_1 = \mathbf{v}_0\nabla\mathbf{v}_0 - (\mathbf{v}_0\nabla)\mathbf{v}_0,\ \nabla\mathbf{u}_1 = 0,\\ (\mathbf{u}_1)_\Sigma = (\mathbf{v}_1)_\Sigma = 0,\ (\mathbf{v}_1)_{S_0} = (\mathbf{u}_1)_{S_0} = -f_1\frac{\partial\mathbf{v}_0}{\partial\mathbf{n}},\\ \left(\mathbf{\Omega}_0\mathbf{\Omega}_1^* + \mathbf{\Omega}_1\mathbf{\Omega}_0 + f_1\frac{\partial}{\partial\mathbf{n}}\Omega_0^2\right)_{S_0} = 2C_1,\ \int\limits_{S_0} f_1 ds = 0.\end{gathered} \tag{1.5}$$

We reduce the number of unknown functions in problem (1.5) by adding the equations for $\mathbf{v}_1$ and $\mathbf{u}_1$ and changing to the notation

$$\mathbf{w}_1 = \frac{1}{2}(\mathbf{v}_1 + \mathbf{u}_1),\quad s_1 = \frac{1}{2}\left(p_1 + q_1 + \frac{v_0^2}{2}\right),\quad \boldsymbol{\omega}_1 = \frac{1}{2}(\mathbf{\Omega}_1 + \mathbf{\Omega}_1^*).$$

This gives

$$\begin{gathered}\Delta\mathbf{w}_1 - \nabla s_1 = 0,\ \nabla\mathbf{w}_1 = 0,\\ (\mathbf{w}_1)_\Sigma = 0,\quad (\mathbf{w}_1)_{S_0} = -f_1\frac{\partial\mathbf{v}_0}{\partial\mathbf{n}},\\ \left(\mathbf{\Omega}\boldsymbol{\omega}_{01} + f_1\frac{\partial}{\partial\mathbf{n}}\Omega_0^2\right)_{S_0} = C_1,\quad \int\limits_{S_0} f_1 dS = 0.\end{gathered} \tag{1.6}$$

It might be noted that since the constant C_1 is determined from the isoperimetric condition, the functions $\mathbf{w}_1$ and f_1 enter the boundary-value problem (1.6) homogeneously. Consequently, problem (1.6) has the trivial solution $\mathbf{w}_1 \equiv 0$, $f_1 \equiv 0$, and therefore $\mathbf{u}_1 = -\mathbf{v}_1$ and $\Omega_1^* = -\Omega_1$.

In the second approximation in Re problem (1.1) takes the form

$$\begin{gathered}\Delta\mathbf{v}_2 - \nabla p_2 = (\mathbf{v}_1\nabla)\mathbf{v}_0 + (\mathbf{v}_0\nabla)\mathbf{v}_1,\ \nabla\mathbf{v}_2 = 0,\\ \Delta\mathbf{u}_2 - \nabla q_2 = (\mathbf{v}_0\nabla)\mathbf{v}_1 + \mathbf{v}_0\nabla\mathbf{v}_1 - (\mathbf{v}_1\nabla)\mathbf{v}_0 - \mathbf{v}_1\nabla\mathbf{v}_0,\ \nabla\mathbf{u}_2 = 0,\\ (\mathbf{v}_2)_\Sigma = (\mathbf{u}_2)_\Sigma = 0,\ (\mathbf{v}_2)_{S_0} = (\mathbf{u}_2)_{S_0} = -f_2\frac{\partial\mathbf{v}_0}{\partial\mathbf{n}},\\ \left(\mathbf{\Omega}_2\mathbf{\Omega}_0 + \mathbf{\Omega}_0\mathbf{\Omega}_2^* - 2\Omega_1^2 + f_2\frac{\partial}{\partial\mathbf{n}}\Omega_0^2\right)_{S_0} = 2C_2,\ \int\limits_{S_0} f_2 dS = 0.\end{gathered}$$

Here it has been taken into account that $f_1 \equiv 0$, $v_0 = u_0$, $v_1 = -u_1$. We decrease the number of equations and unknown functions by adding the equations for $\mathbf{v}_2$ and $\mathbf{u}_2$ and changing to the notation

$$\mathbf{w}_2 = \frac{1}{2}(\mathbf{u}_2 + \mathbf{v}_2),\ s_2 = \frac{1}{2}[p_2 + q_2 - \mathbf{v}_1\mathbf{v}_0],\ \omega_2 = \frac{1}{2}(\Omega_2 + \Omega_2^*).$$

This gives

$$\begin{gathered}\Delta\mathbf{w}_2 - \nabla s_2 = (\mathbf{v}_0\nabla)\mathbf{v}_1 + \mathbf{v}_0\nabla\mathbf{v}_1,\quad \nabla\mathbf{w}_2 = 0,\\ (\mathbf{w}_2)_{S_0} = -f_2\frac{\partial\mathbf{v}_0}{\partial\mathbf{n}},\quad (\mathbf{w}_2)_\Sigma = 0,\\ \left(\Omega_0\omega_2 - \Omega_1^2 + f_2\frac{\partial}{\partial\mathbf{n}}\Omega_0^2\right)_{S_0} = C_2,\ \int_{S_0} f_2 dS = 0.\end{gathered} \tag{1.7}$$

§2. Suppose a uniform translational flow $\mathbf{v}_\Sigma$ = const directed along the x axis is specified on the surface Σ. The surface Σ is symmetric with respect to the yz plane and the midsection of the body S_0 which is optimum in the Stokes approximation and passes through the origin of coordinates. In this case the problem of determining the shape of S_0 admits a solution which is symmetric with respect to the yz plane. We show that if the surface S_0 is symmetric with respect to the yz plane, the function f_2 which is determined in solving problem (1.7) will be an even function of the x coordinate and, consequently, a body which is optimum for nonzero Reynolds numbers will be symmetric with respect to the yz plane to an accuracy of $O(Re^3)$. We introduce the notation

$$\begin{gathered}v_{ix}^+ = \frac{1}{2}[v_{ix}(x, y, z) + v_{ix}(-x, y, z)],\ v_{ix}^- = \frac{1}{2}[v_{ix}(x, y, z) - v_{ix}(-x, y, z)],\\ v_{iy}^+ = \frac{1}{2}[v_{iy}(x, y, z) - v_{iy}(-x, y, z)],\ v_{iy}^- = \frac{1}{2}[v_{iy}(x, y, z) + v_{iy}(-x, y, z)],\\ v_{iz}^+ = \frac{1}{2}[v_{iz}(x, y, z) - v_{iz}(-x, y, z)],\ v_{iz}^- = \frac{1}{2}[v_{iz}(x, y, z) + v_{iz}(-x, y, z)],\\ p_i^+ = \frac{1}{2}[p_i(x, y, z) - p_i(-x, y, z)],\ p_i^- = \frac{1}{2}[p_i(x, y, z) + p_i(-x, y, z)],\\ f_i^+ = \frac{1}{2}[f_i(x, t) + f_i(-x, t)],\ f_i^- = \frac{1}{2}[f_i(x, t) - f_i(-x, t)].\end{gathered} \tag{2.1}$$

Here it is assumed that the surface S_0 is determined by equations of the form $y = y(x, t)$ and $z = z(x, t)$. After substituting Eqs. (2.1) into boundary-value problems (1.4) and (1.5) we obtain $v_0^- = v_0^+ = 0$. Substituting (2.1) into problem (1.7) and taking account of the fact that $\mathbf{v}_0^- = \mathbf{v}_1^+ = 0$ we obtain

$$\begin{gathered}\Delta\mathbf{w}_2^+ - \nabla s_2^+ = (\mathbf{v}_0^+\nabla)\mathbf{v}_1^- + \mathbf{v}_0^+\nabla\mathbf{v}_1^-,\quad \mathbf{w}_2^- = 0,\\ (\mathbf{w}_2^+)_\Sigma = 0,\ (\mathbf{w}_2^+)_{S_0} = -f_2^+\frac{\partial\mathbf{v}_0^+}{\partial\mathbf{n}},\\ \left[\Omega_0^+\omega_2^+ - (\Omega_1^-)^2 + f_2^+\frac{\partial}{\partial\mathbf{n}}(\Omega_0^+)^2\right]_{S_0} = C_2^+,\\ \Delta\mathbf{w}_2^- - \nabla s_2^- = 0,\ \nabla\mathbf{w}_2^- = 0,\ (\mathbf{w}_2^-)_\Sigma = 0,\ (\mathbf{w}_2^-)_{S_0} = -f_2^-\frac{\partial\mathbf{v}_0^+}{\partial\mathbf{n}},\\ \left[\Omega_0^+\omega_2^- + f_2^-\frac{\partial}{\partial\mathbf{n}}(\Omega_0^+)^2\right]_{S_0} = C_2^-,\end{gathered}$$

where the functions with superscripts + and − are defined in analogy with the functions v_i^+, $\mathbf{v}_i^-$, p_i^+, and p_i^-; C_2^+ and C_2^- are constants determined from the isoperimetric condition. Since f_2^- is an odd function of x, the equations

$$\int_{S_0} f_2^- dS = 0,\quad \int_{S_0} f_2^+ dS = 0.$$

must be satisfied. It might be noted that the boundary-value problem for the functions $\mathbf{w}_2^-$, s_2^-, and f_2^- is homogeneous and therefore has the trivial solution $\mathbf{w}_2^- \equiv 0$, $f_2^- \equiv 0$. Thus, it has been shown that boundary-value problem (1.7) has a solution which is symmetric with respect to the yz plane.

§3. In [4, 5] the function G to be minimized was chosen as the rate of dissipation of energy over the whole volume of fluid under investigation

$$G(S)=\int\limits_V \sum_{i,j=1}^{3}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)^2 dV.$$

This functional depends on the shape of the body and on the Reynolds number as on a parameter. Suppose the surface of a certain body S is described by the equations $x_i = X_i(r, t)$. We consider a family of bodies S_Σ with a surface described by the equations $x_i = X_i(r, t) + \varepsilon n_i f(r, t)$, where ε is a parameter and $f(r, t)$ is a fixed function. Then the values of the functional on this family will be a function of two variables $G(S_\Sigma, \mathrm{Re}) = g(\varepsilon, \mathrm{Re})$. We expand the function $g(\varepsilon, \mathrm{Re})$ in a Taylor series in the neighborhood of the point $\varepsilon = 0$, $\mathrm{Re} = 0$. We have

$$g(\varepsilon, \mathrm{Re}) = g(0, 0) + \varepsilon\frac{\partial g}{\partial \varepsilon} + \mathrm{Re}\frac{\partial g}{\partial \mathrm{Re}} + \frac{1}{2}\left(\varepsilon^2\frac{\partial^2 g}{\partial \varepsilon^2} + 2\varepsilon\,\mathrm{Re}\frac{\partial^2 g}{\partial\varepsilon\partial \mathrm{Re}} + \mathrm{Re}^2\frac{\partial^2 g}{\partial \mathrm{Re}^2}\right) + \ldots$$

Here all the derivatives are evaluated at the point $\varepsilon = 0$, $\mathrm{Re} = 0$. If $X_i(r, t) = x_{0i}(r, t)$ it follows from the optimality condition that $\partial g/\partial\varepsilon = 0$. In addition, since a body optimum in the Stokes approximation is optimum also in the first approximation in the Reynolds number, $\partial/\partial\mathrm{Re}(\partial g/\partial\varepsilon) = 0$. Thus for variations of the surface S_0 the contribution to the functional is $^1/_2(\varepsilon^2\partial^2 g/\partial\varepsilon^2)$. Now setting $\varepsilon = \mathrm{Re}^2$ and $f(r, t) = f_2(r, t)$, we find that the family of surfaces S_Σ to an accuracy $O(\mathrm{Re}^3)$ coincides with the family of optimum bodies and therefore for small Reynolds numbers the contribution from optimization is of the order $O(\varepsilon^2) = O(\mathrm{Re}^4)$, and consequently a body which is optimum in the Stokes approximation can, to a high degree of accuracy, be considered optimum also for small Reynolds numbers.

Let us consider two bodies S and $S_1 \supseteq S$. We show that to an accuracy $O(\mathrm{Re}^2)$ that body S_1 has a resistance larger than that of body S. To do this we consider a one-parameter family of bodies $S(\alpha)$ described by the equations $x_i = x_i(r, t, \alpha)$ such that $S(0) = S$, $S(1) = S_1$ and for any $\delta > 0$, $S(\alpha + \delta) \supseteq S(\alpha)$. On this family of bodies the functional will be a function of the single variable α: $G[S(\alpha)] = g(\alpha)$. From the expression for the first variation of the functional G [4, 5] it follows that

$$\frac{\partial g}{\partial\alpha} = \int\limits_{S(\alpha)} f\left(\frac{\partial \mathbf{v}}{\partial \mathbf{n}}\frac{\partial \mathbf{u}}{\partial \mathbf{n}}\right)dS,\quad f = n_i\frac{\partial x_i}{\partial\alpha}.$$

Expanding the functions **v** and **u** in powers of Re we obtain

$$\frac{\partial g}{\partial\alpha} = \int\limits_{S(\alpha)} f\left[\left(\frac{\partial \mathbf{v}_0}{\partial \mathbf{n}}\right)^2 + \mathrm{Re}\left(\frac{\partial \mathbf{v}_0}{\partial \mathbf{n}}\frac{\partial \mathbf{u}_1}{\partial \mathbf{n}} + \frac{\partial \mathbf{u}_0}{\partial \mathbf{n}}\frac{\partial \mathbf{v}_1}{\partial \mathbf{n}}\right)\right]dS + O(\mathrm{Re}^2).$$

The functions $\mathbf{u}_1$ and $\mathbf{v}_1$ must satisfy the zero boundary conditions. Therefore, as is clear from Eqs. (1.4) and (1.5), $\mathbf{u}_0 = \mathbf{v}_0$, $\mathbf{u}_1 = -\mathbf{v}_1$, and therefore

$$\frac{\partial g}{\partial\alpha} = \int\limits_{S(\alpha)} f\left(\frac{\partial \mathbf{v}_0}{\partial \mathbf{n}}\right)^2 dS + O(\mathrm{Re}^2).$$

Since for $\delta > 0$, $S(\alpha + \delta) \supseteq S(\alpha)$, then $f \geq 0$, and therefore $\partial g/\partial\alpha \geq 0$, and since g is a monotonic function, $g(0) \leq g(1)$. Thus, it has been shown that any body containing the given one with an accuracy $O(\mathrm{Re}^2)$ has a resistance larger than that of the given body.

§4. In the above discussions it was essentially assumed that the outer surface Σ is finite, since expansions (1.3) are valid only when $\mathrm{Re}\cdot x_i \ll 1$. In an infinite domain expansions of the form (1.3) must be joined with the outer Oseen expansion ($\mathrm{Re} \to 0$ for $\xi_i = \mathrm{Re}\cdot x_i$ fixed) by replacing the boundary conditions on the surface Σ by appropriate joining conditions. In this case, just as for a finite domain, it can be shown that the function $f_1 \equiv 0$ satisfies the necessary optimality conditions. By changing to the variable $\xi_i = \mathrm{Re}\cdot x_i$ and assuming that a uniform translational flow $\mathbf{v}_\infty = \mathrm{const}$ is specified at infinity we obtain a system of equations and boundary conditions for the functions of the outer expansion

$$\Delta_\xi \mathbf{v}_\xi - \nabla_\xi p = \mathrm{Re}(\mathbf{v}_\xi \nabla_\xi)\mathbf{v}_\xi, \qquad \nabla_\xi \mathbf{v}_\xi = 0,$$
$$\Delta_\xi \mathbf{u}_\xi - \nabla_\xi q = \mathrm{Re}[\mathbf{u}_\xi \nabla_\xi \mathbf{v}_\xi - (\mathbf{v}_\xi \nabla_\xi)\mathbf{u}_\xi], \qquad \nabla_\xi \mathbf{u}_\xi = 0, \tag{4.1}$$
$$(\mathbf{v}_\xi)_\infty = (\mathbf{u}_\xi)_\infty = (1/\mathrm{Re})\mathbf{v}_\infty$$

(the subscript ξ denotes that the corresponding components of the vectors are evaluated in the coordinates ξ_i). Assuming that Re is small we expand the functions $\mathbf{v}_\xi$, p, $\mathbf{u}_\xi$, and q in powers of Re. Since the boundary condition at infinity is of the order Re^{-1}, the expansion of the functions $\mathbf{v}_\xi$, p, $\mathbf{u}_\xi$, and q will start with terms of the order Re^{-1}

$$\mathbf{v}_\xi = \frac{1}{\mathrm{Re}}\mathbf{v}_\xi^0 + \mathbf{v}_\xi^1 + o(1), \quad p = \frac{1}{\mathrm{Re}}p^0 + p^1 + o(1),$$
$$\mathbf{u}_\xi = \frac{1}{\mathrm{Re}}\mathbf{u}_\xi^0 + \mathbf{u}_\xi^1 + o(1), \quad q = \frac{1}{\mathrm{Re}}q^0 + q^1 + o(1). \tag{4.2}$$

Substituting expansion (4.2) into the equations and boundary conditions (4.1), equating coefficients of equal powers of Re, and writing the condition for joining with the inner expansion (1.3), we obtain a sequence of boundary-value problems for determining the functions $\mathbf{v}_\xi^1$, p^1, $\mathbf{u}_\xi^1$, and q^1. It can be seen that the solution for the functions $\mathbf{v}_\xi^0$ and $\mathbf{u}_\xi^0$ will be $\mathbf{v}_\xi^0 = \mathbf{u}_\xi^0 = \mathbf{v}_\infty$. For functions of the first approximation $\mathbf{v}_\xi^1$, p^1, $\mathbf{u}_\xi^1$, and q^1 Eqs. (4.1) are transformed into the Oseen equations

$$\Delta_\xi \mathbf{v}_\xi^1 - \nabla_\xi p^1 = (\mathbf{v}_\infty \nabla_\xi)\mathbf{v}_\xi^1, \quad \Delta_\xi \mathbf{u}_\xi^1 - \nabla_\xi q^1 = -(\mathbf{v}_\infty \nabla_\xi)\mathbf{u}_\xi^1, \quad \nabla_\xi \mathbf{u}_\xi^1 = \nabla_\xi \mathbf{v}_\xi^1 = 0.$$

In order to join the functions $\mathbf{v}_\xi^1$, p^1, $\mathbf{u}_\xi^1$, and q^1 with the corresponding functions of expansion (1.3) we join the functions $\mathbf{w}_\xi^1 = {}^1/_2(\mathbf{v}_\xi^1 + \mathbf{u}_\xi^1)$, $s^1 = {}^1/_2(p^1 + q^1 + v_\infty^2/2)$ with the functions $\mathbf{w}_1$ and s_1. For $\mathbf{w}_\xi^1$ and s^1 we have Stokes equation

$$\Delta_\xi \mathbf{w}_\xi^1 - \nabla_\xi s^1 = 0, \quad \nabla_\xi \mathbf{w}_\xi^1 = 0, (\mathbf{w}^1)_\infty = 0. \tag{4.3}$$

The joining condition in this case has the form

$$\lim_{\mathrm{Re}\to 0} \mathbf{w}_x^1 = \lim_{\mathrm{Re}\to 0} \mathbf{w}_1 \tag{4.4}$$

where x_i and ξ_i are fixed.

It can be seen that the solution of problem (1.6) and (4.3) for joining condition (4.4) is $f_1 \equiv 0$, $\mathbf{w}_1 \equiv 0$, and $\mathbf{w}_\xi^1 \equiv 0$. Thus, it is clear that for an infinite domain the necessary optimality conditions are satisfied to an accuracy O(Re) on body $S_\bullet$, which is optimum for zero Reynolds numbers. In the second approximation terms of the order $v_2 = O(\ln \mathrm{Re})$ arise for $\mathbf{v}$ as a result of joining expansion (1.3) with the Oseen expansion (cf. e.g. [6]), and therefore the function f_2 will also be of the order $f_2 = O(\ln \mathrm{Re})$. The same terms arise in the functional G also. In this case by carrying through arguments similar to those in Sec. 3 it can be shown that the change in the functional as a result of optimization is of the order $O(\mathrm{Re}^3)$.

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